INSPECTION AND SELECTION OF REPRESENTATIONS

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26 June 2019

OVERVIEW

Motivation

THE BIG PICTURE

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People change representation to get to information

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We aim to build a tool that can help choose a suitable representation for a given problem for a particular person

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SOLUTION UNDER BAYESIAN APPROACH

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$$\Pr(\bar{b}) = \Pr(\bar{b} \cap \bar{f}) + \Pr(\bar{b} \cap f) \tag{1}$$

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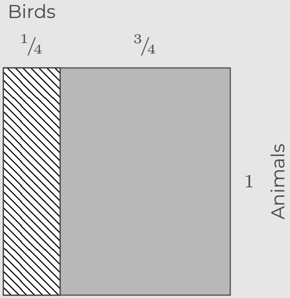

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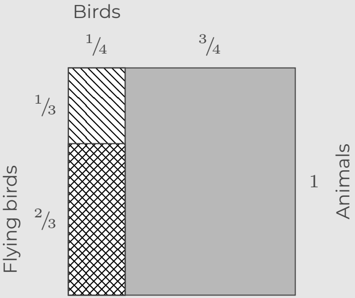

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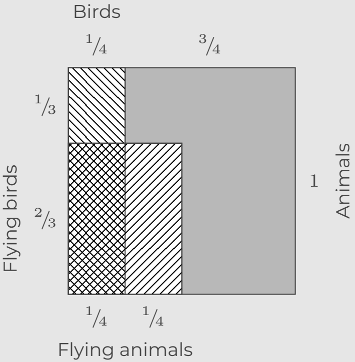

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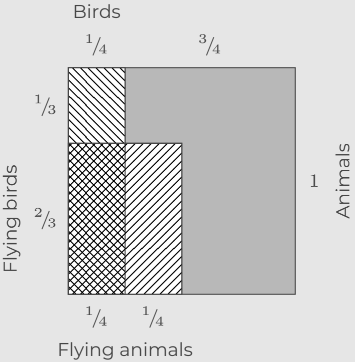

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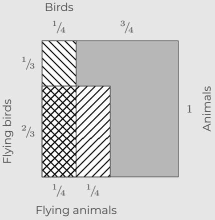

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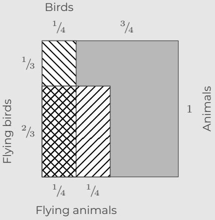

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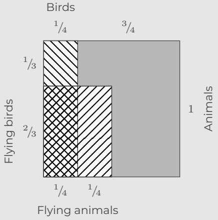

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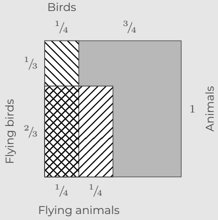

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They make different things evident.
They require different kinds of knowledge.
They allow different manipulations.

How to describe representations?

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By the *symbols* they use, the *inferences* that can be done, and the *knowledge* they encode.

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For each RS, encode this into a table.
This is our knowledge of the RS.

Bayesian

kind	value
types	real, event
tokens	Ω, $\emptyset$, 0, 1, $=$, $+$, $-$, $*$, $\div$, $\cup$, $\cap$, $\setminus$, $^{-}$, $\Pr$, $\mid$
patterns	$\_ : \texttt{real} = \_ : \texttt{real}$, $\Pr(\_ \mid \_) = \_$, ...
facts	Bayes' theorem, law of total probability, non-negative probability, ...
tactics	rewrite, arithmetic calculation

Geometric

kind	value
types	point, segment, region, real, string
tokens	\$point, \$segment, \$shade
patterns	\$rectangle, \$contained, ...
facts	scale-independence of ratio, non-negative area, area additivity, ...
tactics	draw point, draw segment, delete, join, compare sizes

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BUT...
Not everything is equally important!

PROBLEMS AND THEIR IMPORTANCE HIERARCHY

	kind	value
importance ↑	error allowed	0
	answer type	ratio
	tokens	probability, and, not
	types	ratio, class
	patterns	_ : ratio of _ : class are _ : class, . . .
	facts	Bayes’ theorem, law of total probability,. . .
	tactics	deduction, calculation
	tokens	one, quarter, all, animal, birds, two, thirds, can, fly, half, flying, X, cannot
	related tokens	times, divided_by, plus, minus, equals, or, union, intersection, . . .
	# of tokens	67
	# of distinct tokens	31
noise	tokens	feathers
	related tokens	beast, animate, creature, . . .

How to select representations?

CORRESPONDENCES

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Mostly analogical representational matches between tokens, types, patterns, tactics, facts etc.

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Problem in NL	Bayesian RS	Geometric RS
is about *classes*	represents *events*	represents *regions*
is about *probability*	represents $\Pr$	represents *area* (size)
is about *ratios*	represents *real numbers*	represents *real numbers*
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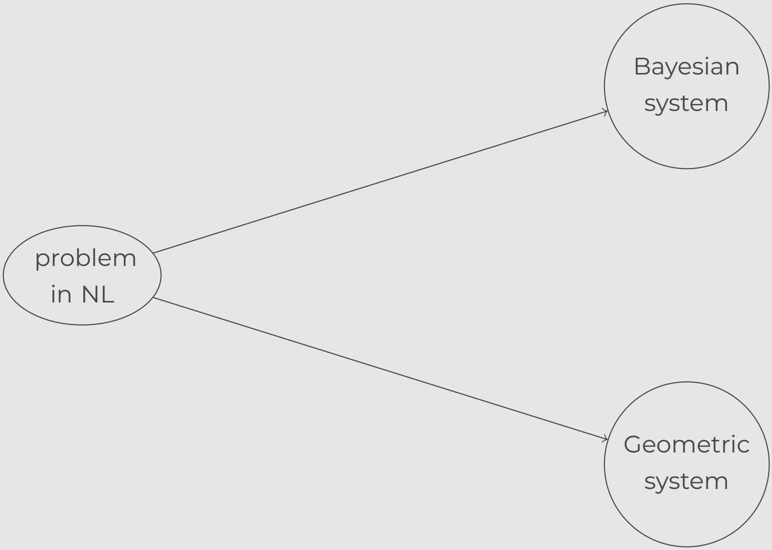
In practice, we build correspondence tables to relate pairs of properties with a *score* (how good a reason is it?)

CORRESPONDENCE TABLES



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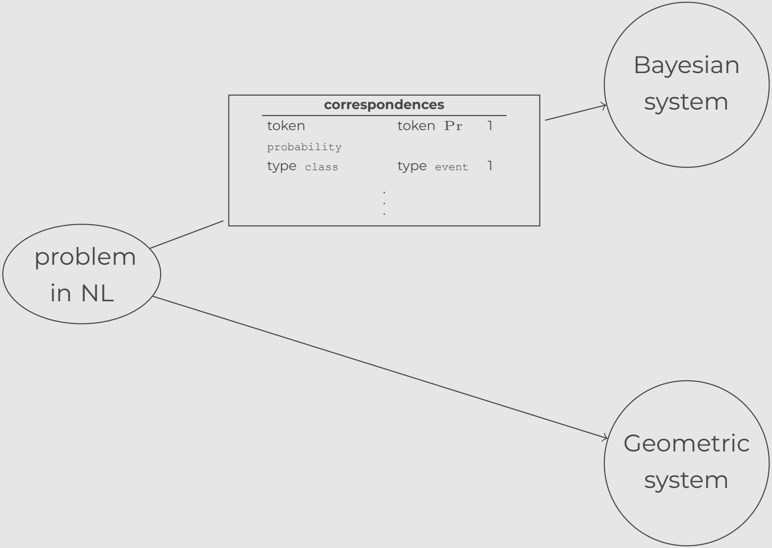

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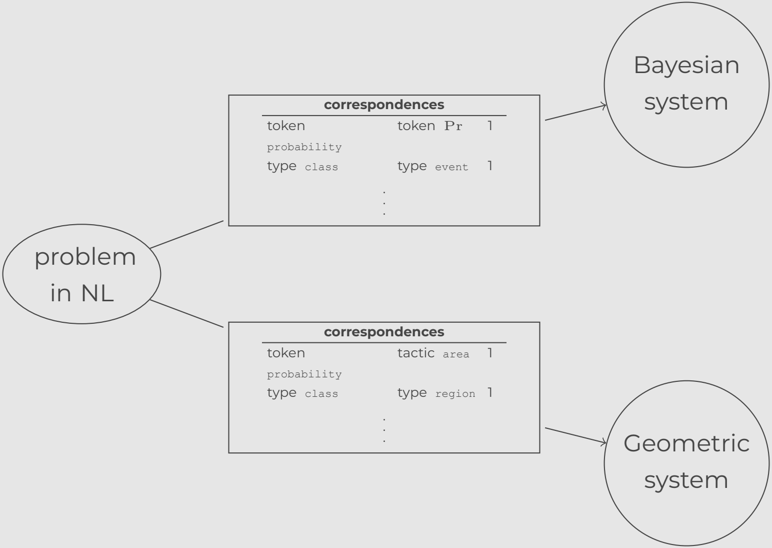

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MAKING A RECOMMENDATION

Bayesian	9.3
Geometric	7.2
Natural Language	6.9
Contingency	5.4
Euler	1.5

Conclusions and work in progress

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A framework for representing representations

A proof-of-concept algorithm for suitability

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How can we evaluate this?

WHAT ARE WE WORKING ON NOW?

How to formalise the concept of correspondence.

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Cognitive properties of representations.

- Figuring out what how to calculate cognitive costs
- How to work the user into the calculations?

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